HEAT EXCHANGE NEAR A FRONT POINT WITH STEPWISE VARIATION OF THE STREAM TEMPERATURE

V. L. Sergeev and A. S. Strogii

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The solution is given for the conjugate nonsteady problem of heat exchange in the region of the stagnation point of an axisymmetric body with spherical blunting with stepwise variation of the stream temperature.

Upon the insertion of a body into a stream having a temperature different from the temperature of the body the following scheme of the process of nonsteady heat exchange is possible. If one neglects the heating (cooling) of the thin layer of liquid (gas) adjacent to the body, then one can assume that at the initial time there was a temperature jump in the entire volume of liquid down to the surface, and then equalization of the temperatures in the body and the stream occurs. The problem consists in determining the characteristics of the heat exchange in the region of the stagnation point of an axisymmetric body with spherical blunting under these nonsteady conditions.

Published experimental data [1, 2] indicate a possible dependence of the coefficient of heat exchange under nonsteady conditions on time, the thermophysical properties of the body, and its characteristic size in the direction of heat flow. In other reports [3, 4] such a dependence was not detected or its possibility is denied.

In the theoretical solution of such problems, one usually assumes [5, 6] that the surface temperature of the body remains constant in the process of nonsteady heat exchange, which holds in certain limiting cases. According to the data of [5, 6], the characteristics of nonsteady heat exchange can depend on time and the properties of the fluid and the material of the body. In [6] it is concluded that nonsteady heat exchange has a quasisteady character for the typical experimental conditions in shock tubes and other similar devices. In [5, 6] a subsonic, laminar, steady stream is considered and the properties of the fluid are assumed to be constant.

With allowance for the enumerated assumptions, the problem has the following formulation:

$$\frac{\partial (xu)}{\partial x} + \frac{\partial (xv)}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \beta^2 x + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial t}{\partial \tau^*} + u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = a_{\mathrm{f}}\frac{\partial^2 t}{\partial y^2}, \tag{3}$$

$$\frac{\partial t}{\partial \tau^*} = a_{\mathrm{b}}\frac{\partial^2 t}{\partial y^2}, \tag{4}$$

$$t(x,\ y,\ \tau^* < \tau_1^*) = t_\infty, \tag{5}$$

$$t_{\mathrm{b}}(x,\ y,\ \tau^* = \tau_1^*) = t_0, \tag{6}$$

$$t_{\mathrm{f}}(x,\ 0,\ \tau^* > \tau_1^*) = t_{\mathrm{b}}(x,\ 0,\ \tau^* > \tau_1^*),$$

$$\lambda_{\mathrm{f}}\left[\frac{\partial t_{\mathrm{f}}(x,\ 0,\ \tau^* > \tau_1^*)}{\partial y}\right] = \lambda_{\mathrm{b}}\left[\frac{\partial t_{\mathrm{b}}(x,\ 0,\ \tau^* > \tau_1^*)}{\partial y}\right], \tag{7}$$

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$$t_f\ (x,\ \infty,\ \tau^*) = t_\infty, \tag{8}$$

$$\frac{\partial t_b(x,\ -\delta,\ \tau^*)}{\partial y} = 0. \tag{9}$$

Using the stream function $\Psi = (\nu\beta/2)^{1/2}(x/D)xf$ [5], where the variable f is connected with the velocity components by the relations $u = Bxf'$ and $v = -(2\nu\beta)^{1/2}f$, we can satisfy the continuity equation. The derivative $\partial t/\partial x$ in the energy equation (3) is usually neglected [5, 6] on the grounds that at the stagnation point it is equal to zero by symmetry. We assume that in the region of the stagnation point the body consists of a hollow sphere of thermally conducting material of thickness δ, and the temperature drop over its thickness can be neglected. Such conditions are realized in experiment. This condition allows us to replace the equation of heat conduction of the wall by the expression

$$q = \delta c\rho dt/d\tau^*, \tag{10}$$

which allows for the time variation of the surface temperature of the wall. In Eq. (10) q is the specific heat flux at the surface, $c\rho$ is the volumetric heat capacity, and τ^* is the time. The heat flux at the inner surface of the wall is assumed to equal zero. We assume that heat transfer in the body is one-dimensional. This condition is usually satisfied in heat-flux probes [1, 2].

With allowance for the above comments, the formulation of this problem in dimensionless form includes the momentum equation

$$f''' + ff'' + \frac{1}{2}(1 - f'^2) = 0, \tag{11}$$

the solution of which, with allowance for the boundary conditions $f(0) = f'(0) = 0$ and $f'(\infty) = 1$, is [5]

$$f(\eta) \cong 0.46385\,\eta^2 - 0.08333\,\eta^3 + 0.0006442\,\eta^6 - 0.0000496\,\eta^7 - 0.00001186\,\eta^9; \tag{12}$$

the energy equation

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\eta^2} + \Pr f\frac{\partial\theta}{\partial\eta}, \tag{13}$$

where $\theta = (t - t_\infty)/(t_0 - t_\infty)$; $\tau = 2\beta\tau^*/\Pr$; $\eta = y(2\beta/\nu)^{1/2}$; β is the velocity gradient at the stagnation point, equal to $3U_\infty/2R$ for a sphere; and the boundary conditions

$$\tau < \tau_1, \quad \theta = 0, \tag{14}$$

$$\tau = \tau_1, \quad \theta_b = 1, \tag{15}$$

$$\tau > \tau_1, \ \theta_{f,\eta=0} = \theta_{b,\eta=0}, \quad -\left(\frac{\partial\theta_f}{\partial\eta}\right)_{\eta=0} = \sqrt{6\,\mathrm{Re}}\,\frac{\delta}{D}\,k_{c\rho}\frac{d\theta_b}{d\tau}, \tag{16}$$

$$\theta_{f,\eta=\infty} = 0, \tag{17}$$

where $\mathrm{Re} = \beta D^2/3\nu$ for a sphere; $k_{c\rho} = (c\rho)_b/(c_p\rho)_f$; parameters with the subscript b refer to the body and those with f to the fluid.

The unknown quantity is the Nusselt number Nu, defined by the dependence

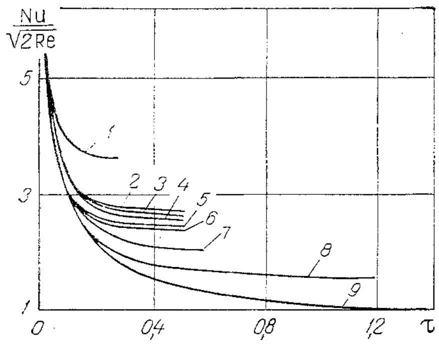
$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = -\frac{\sqrt{6}}{\theta_b}\left(\frac{\partial\theta_f}{\partial\eta}\right)_{\eta=0}. \tag{18}$$

From (11) and (13)-(18) it follows that

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \varphi\,(\Pr,\ \mathrm{Re},\ \delta/D,\ (c\rho)_b/(c_p\rho)_f,\ \beta\tau^*), \tag{19}$$

i.e., besides the usual parameters such as the Prandtl and Reynolds numbers Pr and Re, Nu depends on the dimensionless wall thickness, the ratio of the products of the specific heat times the material density of the body and the fluid, and the dimensionless time. The influence of the ratio of the coefficients of thermal conductivity of the body and the fluid times the complex $\mathrm{Nu}/\sqrt{\mathrm{Re}}$, which exists under actual conditions, was investigated in [7].

The calculations were made on a computer for ranges of Pr from 0.015 to 50, Re from 10^2 to 10^6, δ/D from 0.001 to 0.2, and $k_{c\rho}$ from 0.3 to 0.9 for Pr > 2 and from $2\cdot10^3$ to $6\cdot10^3$



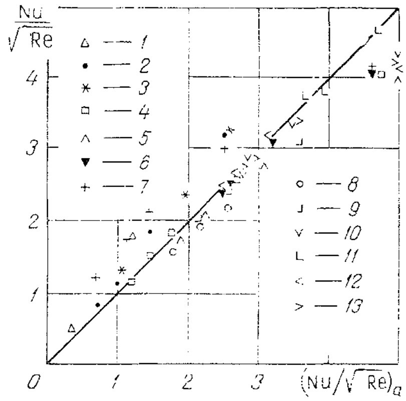


Fig. 1 Fig. 2

Fig. 1. Examples of the dependence of the complex $Nu/\sqrt{2Re}$ on the dimensionless time: 1, 4, 7, 8, 9) Re = 10^3, δ/D = 0.2, $k_{c\rho}$ = 0.3; 1) Pr = 50; 4) 20; 7) 10; 8) 5; 9) 0.7; 2, 3, 5) Pr = 20, δ/D = 0.2; 2) Re = 10^6, $k_{c\rho}$ = 0.3; 3) Re = 10^3, $k_{c\rho}$ = 0.9; 5) Re = 10^2, $k_{c\rho}$ = 0.3; 6) Pr = 20, $k_{c\rho}$ = 0.3; Re = 10^3, δ/D = 0.05.

Fig. 2. Dependence of $Nu/\sqrt{Re}$ on the complex $(Nu/\sqrt{Re})_a$, calculated from Eqs. (23) and (24): 1-11) Re = 10^3; 1) Pr = 5, δ/D = 0.01; 2) 10, 0.01; 3) 20, 0.01; 4) 5, 0.05; 5) 10, 0.05; 6) 20, 0.05; 7) 2, 0.2; 8) 5, 0.2; 9) 10, 0.2; 10) 20, 0.2; 11) 50, 0.2; 12, 13) Pr = 20, δ/D = 0.2; 12) Re = 10^2; 13) 10^6.

for Pr $\leqslant$ 0.7. The values of the parameter $k_{c\rho}$ correspond to combinations of bodies made of metals or certain insulators with streams of gases, water, and certain more viscous liquids.

The calculations showed that the complex $Nu/\sqrt{Re}$ essentially depends on time, the Prandtl number, and the ratio δ/D and to a lesser extent on Re and $k_{c\rho}$ (Fig. 1). The function Nu ∿ $Re^{0.5}$, characteristic of steady heat exchange in laminar flow, will be different under nonsteady conditions. With increasing time $Nu/\sqrt{Re}$ decreases, approaching a constant (quasisteady) value. An increase in Pr and δ/D leads to an increase in $Nu/\sqrt{Re}$. The quasisteady value of $Nu/\sqrt{Re}$ grows with an increase in δ/D and Re, approaching a constant value.

An approximation of the limiting dependence of $Nu/\sqrt{Re}$ on Pr for δ/D $\geqslant$ 0.2, Re $\geqslant 10^5$, and $k_{c\rho}$ = 0.3-0.9 leads to the expression

$$Nu/\sqrt{Re} = 1.38\, Pr^{0.35}, \tag{20}$$

which is in satisfactory agreement with the dependence for the case of steady heat exchange [8],

$$Nu/\sqrt{Re} = 1.32\, Pr^{0.4}. \tag{21}$$

Equations (20) and (21) approximate the results of numerical calculation for the values of Pr = 0.7-50 and 0.7-2, respectively.

Under nonsteady conditions with Pr < 1 the heat exchange has a peculiarity, consisting in the fact that for Re = 10^2-10^6, $k_{c\rho}$ = 0.3-6·10^3, δ/D = 0.01-0.3, and Pr = 0.015-1 the enumerated parameters hardly affect the dependence of $Nu/\sqrt{Re}$ on the dimensionless time. For values of τ from 0.025 to 1.4, when the variation of $Nu/\sqrt{Re}$ with time has practically ceased, this dependence is approximated with an error of less than 5% by the expression

$$\frac{Nu}{\sqrt{Re}} = 1{,}34\,[1 - \exp(-0{,}15 - 1{,}96\,\tau)]^{-1}. \tag{22}$$

At τ < 0.025, $Nu/\sqrt{Re}$ = 0.18/τ.

The following equations can be used to determine the value of the complex $Nu/\sqrt{Re}$ in the range of variation of Pr from 2 to 50, of δ/D from 0.01 to 0.2, of Re from 10^2 to 10^6, and of $k_{c\rho}$ from 0.3 to 0.9:

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}=\left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_{\mathrm{qu.st}}\{1-\exp[-1.8(1-e^{-120\delta/D})\mathrm{Pr}^{0.72}\tau]\}^{-1}, \quad (23)$$

$$\left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_{\mathrm{qu.st}}=1.44\,\mathrm{Pr}^{0.35}\left\{1-\exp\left[-52.5\frac{\delta}{D}+52.5\frac{\delta}{D}\exp(-0.157\,\mathrm{Pr}+0.25)\right]\right\}(1-1.59\,\mathrm{Re}^{-0.54}). \quad (24)$$

The corrections to the values of $Nu/\sqrt{Re}$ calculated from these equations can be estimated with the help of Fig. 2.

The calculations qualitatively confirm the dependence, discovered in the experiments of [1, 2], of the coefficient of heat exchange under nonsteady conditions on time, the thermophysical properties of the material of the body, and the wall thickness.

An estimate of the time of significant variation of the complex $Nu/\sqrt{Re}$ gives a value on the order of 10^{-5} sec for air with $R = 10^{-2}$ m and $U_\infty = 300$ m/sec, while for water with the same radius and $U_\infty = 1$ m/sec it gives 10^{-2} sec, which is considerably less than the time of variation of the heat-exchange characteristics found from experiment. The variations of the heat-exchange coefficient with time observed in experiments cannot be explained by relaxation of the boundary layer after a stepwise change in the stream temperature. In nonsteady heat-exchange processes with a characteristic time on the order of that given above, the heat-exchange intensity can differ from the quasisteady value, and the heat-exchange characteristics will depend not only on the stream parameters and the properties of the gas but also on the thermophysical properties and thickness of the body with which the stream interacts.

It also follows from the results obtained that periodic stepwise variation of the stream temperature (or some approximation to it) may be an efficient means of controlling the heat-exchange intensity, permitting a considerable increase in it.

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